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higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

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AUGUST 2011

NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

1 August (X-Paper)
09:00 – 12:00

This question paper consists of 7 pages and a 5-page formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Questions may be answered in any order, but subsections of questions must be kept together.
4. Show ALL the intermediate steps. Simplify where possible.
5. ALL graph work must be done in the ANSWER BOOK.
6. Questions must be answered in blue or black ink.
7. Number the answers correctly according to the numbering system used in this question paper.
8. Write neatly and legibly.

QUESTION 1

1.1 Determine the values of the following limits:

1.1.1 $\lim_{x\to 7}\left(\frac{49-x^2}{x-7}\right)$ (2)

1.1.2 $\lim_{x\to 1}\left(\frac{x^2-1}{\ell nx}\right)$ (3)

1.2 Determine whether the function $f(x)=\frac{x-1}{(2x-\pi)^2}$ is continuous at $x=\frac{\pi}{2}$. (2)

[7]

QUESTION 2

2.1 Determine $\frac{dy}{dx}$ of the following, by making use of the derivatives of $\sin x$ and $\cos x$, as well as the rules of differentiation:

$$y=\tan x$$ (3)

2.2 2.2.1 Make a neat sketch of $y = arc\ \cos x$ for the range $[0;\pi]$. (2)

2.2.3 Derive a formula to determine $\frac{dy}{dx}$ if $y = arc\ \cos x$. (4)

2.3 Determine $\frac{dy}{dx}$ in each of the following cases: (Simplification is NOT required.)

2.3.1 $y=\left[arc\ \sin\left(e^{-x}.x^{-2}\right)\right]^4$ (4)

2.3.2 $y=x^4.\ell nx$ (2)

2.3.3 $y=e^{\left(\frac{\ell nx}{2^x}\right)}$ (3)

2.4 Determine $\frac{dy}{dx}$ if $y=(\sin x)^{e^x}$ with the aid of logarithmic differentiation. (4)

2.5 Determine $\frac{dy}{dx}$ of the implicit function $\cos y - x = y^3$. (3)

[25]

QUESTION 3

3.1 A closed rectangular tank is constructed to occupy $70\ dm^3$ and its length is thrice its breadth. Calculate the height that will minimise the quantity of material to be used.

HINT: First draw the rectangular tank and then develop the formulas for volume and area. (6)

3.2 A conical shaped ice artwork is exposed to the sun. The volume of this cone decreases at a rate of $100\ cm^3.s^{-1}$ and the radius decreases at a rate of $1\ cm.s^{-1}$. Calculate the rate at which the height changes when the height is $10\ cm$ and the radius is $3\ cm$.

HINT: $V = \frac{1}{3}\pi r^2 h$ (4)

3.3 GIVEN:

$$f(x) = -\frac{1}{2}x^3 - \frac{1}{4}x^2 + 2x - 1$$

3.3.1 Determine the coordinates of the turning points of $f(x)$. (2)

3.3.2 Draw up a table of values of x and $f(x)$, with x ranging from $x = -3$ to $x = 3$. (2)

3.3.3 Draw a neat graph of $f(x)$ between these values and show the turning points on it. (2)

3.3.4 Use the table and the graph to estimate a value for the root between $x = 1$ and $x = 2$ of the equation $-\frac{1}{2}x^3 - \frac{1}{4}x^2 + 2x - 1 = 0$ and then use Taylor's/Newton's method to determine a better approximation of this root (root correct to THREE decimal figures). (3)

[19]

QUESTION 4

4.1 Determine the integrals in each of the following cases:

4.1.1 $\int x^{-3}.\ell n\left(\sqrt{x}\right)dx$ (4)

4.1.2 $\int\left[\dfrac{\sin\left(e^{x}+\ell nx\right)}{\left(e^{x}+\dfrac{1}{x}\right)^{-1}}\right].dx$ (3)

4.1.3 $\int\cos^{2}\left(\dfrac{\pi}{x}\right).x^{-2}.dx$ (3)

4.1.4 $\int\sqrt{\sin x}.\sin x.\cos x.dx$ (3)

4.1.5 $\int\dfrac{\sec^{2}x.\tan x}{\sec\left(e^{\ell nx}\right)}.dx$ (3)

4.1.6 $\int\dfrac{1}{2x^{2}+8}.dx$ (3)

4.2 Determine $\int y.dx$ by resolving the integrand into partial fractions:

$$y=\frac{x+1}{(7-x)^{2}}$$

(5)

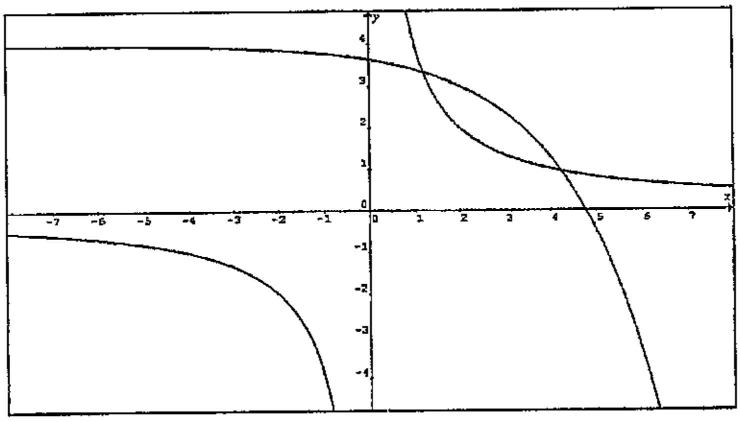
[24]

QUESTION 5

5.1 Evaluate the definite integral: $\int_{e}^{\ell ne} \frac{1}{x}(\ell nx - 4).dx$ (4)

5.2 The sketch below shows the area bounded by the curves $y = -e^{\frac{1}{2}x-1} + 4$ and $xy = 4$.

The intersection points between the two curves $y = -e^{\frac{1}{2}x-1} + 4$ and $xy = 4$ are given as $(1{,}202;3{,}329)$ and $(4{,}234;0{,}945)$.



5.2.1 Draw the sketch in the ANSWER BOOK and show on the sketch the enclosed area and the representative strip. (2)

5.2.2 Calculate the magnitude of the area in QUESTION 5.2.1. (3)

5.2.3 Calculate the volume of solid revolution formed when the area in QUESTION 5.2.2 rotates about the $x-axis$. (4)

5.3 Calculate the second moment of area of a circular lamina with a radius r about an axis through the centre of the lamina and perpendicular to the plane of the lamina. (4)

[17]

QUESTION 6

6.1 Solve the differential equation:

$dy - 3e^{3x}.dx - 9.dx = 0$ (3)

6.2 Determine the particular solution of $-x\dfrac{d^2y}{dx^2} = x.e^{2x} + \dfrac{1}{x^2} + x$, given that $\dfrac{dy}{dx} = 0$, $y = 1$ and $x = -1$. (5)

[8]

TOTAL: 100

MATHEMATICS N5

FORMULA SHEET

Any applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = \frac{1}{2} [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\operatorname{cosec} x}$; $\cos x = \dfrac{1}{\sec x}$

PTO

BINOMIAL THEOREM

$$(x+h)^n = x^n + nx^{n-1}\,h + \frac{n(n-1)}{2!}x^{n-2}\,h^2 + \ldots$$

DIFFERENTIATION

$$e = -\frac{f(a)}{f'(a)}$$

$$r = a + e$$

PRODUCT RULE

$$y = u(x)\,.\,v(x)$$

$$\frac{dy}{dx} = u\,.\,\frac{dv}{dx} + v\,.\,\frac{du}{dx}$$

$$= u \cdot v' + v \cdot u'$$

QUOTIENT RULE

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v\,.\,\frac{du}{dx} - u\,.\,\frac{dv}{dx}}{v^2}$$

$$= \frac{v \cdot u' - u \cdot v'}{v^2}$$

CHAIN RULE

$$y = f(u(x))$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
ax^n	nax^{n-1}	$\frac{ax^{n+1}}{n+1}+c$
a	0	$ax+c$
e^x	e^x	e^x+c
a^x	$a^x.\ln a$	$\frac{a^x}{\ln a}+c$
$\log_e x$	$\frac{1}{x}$	—
$\log_a x$	$\frac{1}{x\ln a}$	—
$\sin x$	$\cos x$	$-\cos x+c$
$\cos x$	$-\sin x$	$\sin x+c$
$\tan x$	$\sec^2 x$	$\ln(\sec x)+c$
$\cot x$	$-\mathrm{cosec}^2 x$	$\ln(\sin x)+c$
$\sec x$	$\sec x\cdot\tan x$	$\ln[\sec x+\tan x]+c$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x\cdot\cot x$	$\ln(\mathrm{cosec}\, x-\cot x)+c$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	—
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	—
$\tan^{-1} x$	$\frac{1}{1+x^2}$	—
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	—
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$	—
$\mathrm{cosec}^{-1} x$	$\frac{-1}{x\sqrt{x^2-1}}$	—

PTO

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
$\frac{1}{\sqrt{a^2 - x^2}}$	—	$\sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{a^2 + x^2}$	—	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{x\sqrt{x^2 - a^2}}$	—	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$
$\sqrt{a^2 - x^2}$	—	$\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2} + c$
$\frac{1}{x^2 - a^2}$	—	$\frac{1}{2a} ln\left(\frac{x - a}{x + a}\right) + c$
$\frac{1}{a^2 - x^2}$	—	$\frac{1}{2a} ln\left(\frac{a + x}{a - x}\right) + c$

INTEGRATION

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + c$$

$$\int [f(x)]^n f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

$$\frac{f(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

$$\frac{f(x)}{(x+a)^n} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3} + \cdots \frac{Z}{(x+a)^n}$$

APPLICATIONS OF INTEGRATION

AREAS

$$A_x = \int_a^b y\,dx\,;\; A_x = \int_a^b (y_1 - y_2)\,dx$$

$$A_y = \int_a^b x\,dy\,;\; A_y = \int_a^b (x_1 - x_2)\,dy$$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \; ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx$$

$$V_y = \pi \int_a^b x^2 dy \; ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \; ; I_y = \int_a^b r^2 dA$$

MOMENTS OF INERTIA

Mass = density × volume

$M = \rho V$

DEFINITION: $I = m r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$